DIGITAL COMPUTER CALCULATION OF TEMPERATURE FIELDS PRODUCED IN A WORKPIECE BY REPEATED APPLICATIONS OF A SOURCE OF HEATING

Yu. A. Mel'nikov and A. K. Tsokur

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A method of digital computer calculation of the temperature fields produced in a component under conditions of repeated heating and cooling is described. The temperature fields produced by internal grinding are calculated.

Various engineering methods of calculation are known, by means of which the thermal and residual stresses produced in a component during machining can be calculated and the possible occurrence of structural changes investigated. These methods presuppose knowledge of the temperature fields produced in the components.

Although there are many papers of an applied character on this subject [1-3] they all adopt a computational scheme involving a single application of the source of heat to the component.

There are many engineering processes in which the component is heated and cooled in a periodic manner. For example, in a real grinding process the component is periodically heated in the cutting zone and cooled outside this zone. The formulas given in the cited papers will reflect the true picture only provided the component manages to lose all its heat during the cooling period and provided the boundary conditions do not change. There is much experimental evidence that the thermal processes do not have time to stabilize themselves during a single revolution of the workpiece even when steps are taken to cool it. In grinding practice it very often happens that the workpiece is machined without any cooling at all (internal grinding, pointing, and so on).

Evidently, a calculation of the temperature field which is based upon a single application of the source of heat cannot give the true picture of the temperature distribution in such cases.

An attempt is made in the present article to develop a computational scheme suitable for repeated applications of the source of heat. The results show that in some cases the single-application scheme predicts temperatures that are much too low, sometimes distorts the temperature fields produced in the component, and fails to give reliable estimates of the maximum temperatures produced – something absolutely essential in the investigation of possible structural changes. Also, computations based on the single-application scheme do not yield the true temperature gradients.

The kinematics of grinding imply that during the machining process each point of the workpiece is periodically heated (when it comes into contact with the abrasive disc) and cooled (in the period between contacts). Each of the cycles of heating and cooling is characterized by its own boundary conditions relating to the thermal state of the component.

We consider the following boundary problem of thermal conductivity:

cycle I (grinding)

$$\frac{\partial u_1(x,\ t)}{\partial t} = a\,\frac{\partial^2 u_1(x,\ t)}{\partial x^2} \quad \left\{\begin{matrix} 0 \leqslant x < \infty \\ 0 \leqslant t < t_{gr} \end{matrix}\right\};$$

$$u_1(x,\ 0) = 0;$$

$$\lambda\,\frac{\partial u_1(0,\ t)}{\partial x} + q = 0; \quad u_1(\infty,\ t) = 0;$$

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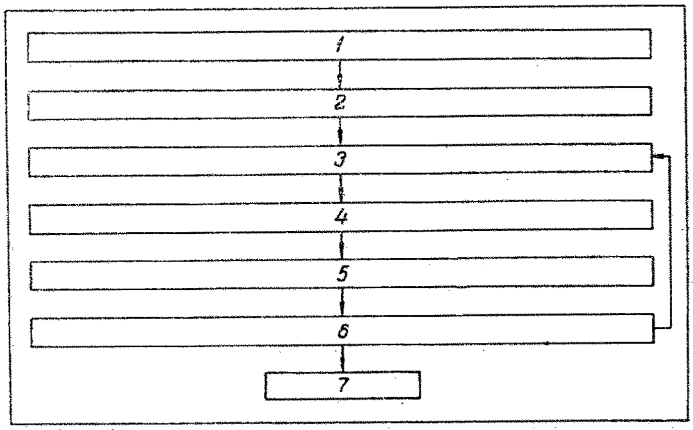

Fig. 1

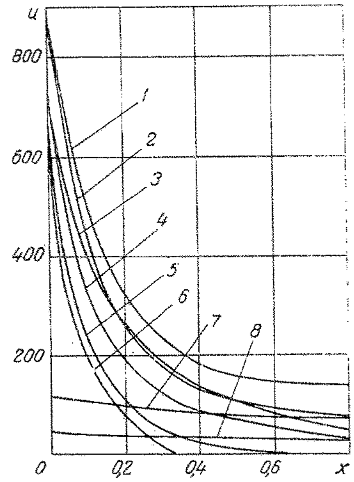

Fig. 2

Fig. 1. Block diagram of program for computing temperature fields on the Minsk-22 digital computer. 1) Input of initial data and printing of symbol characterizing variant; 2) computation of time step and of mesh dimensions for grinding and cooling stages; 3) computation of temperatures at mesh nodes during grinding stage; 4) transfer of temperature values of last time sheet of previous operator to zeroth time sheet of subsequent operator; 5) computation of temperature values at mesh nodes during cooling stage; 6) logical comparison of previous heating-cooling stages with subsequent; 7) stop.

Fig. 2. Temperature fields produced by internal grinding. Units: u (°C), x (mm).

cycle II (cooling)

$$\frac{\partial u_2(x,\ t)}{\partial t} = a\ \frac{\partial^2 u_2(x,\ t)}{\partial x^2} \quad \left\{\begin{matrix} 0 \leqslant x < \infty \\ 0 \leqslant t < t_{\mathrm{cool}} \end{matrix}\right\};$$

$$u_2(x,\ 0) = u_1(x,\ t_{\mathrm{gr}});$$

$$\lambda\ \frac{\partial u_2(0,\ t)}{\partial x} - \alpha u_2(0,\ t) = 0; \quad u_2(\infty,\ t) = 0;$$

cycle III (grinding)

$$\frac{\partial u_3(x,\ t)}{\partial t} = a\ \frac{\partial^2 u_3(x,\ t)}{\partial x^2} \quad \left\{\begin{matrix} 0 \leqslant x < \infty \\ 0 \leqslant t < t_{\mathrm{gr}} \end{matrix}\right\};$$

$$u_3(x,\ 0) = u_2(x,\ t_{\mathrm{cool}});$$

$$\lambda\ \frac{\partial u_2(0,\ t)}{\partial x} + q = 0; \quad u_3(\infty,\ t) = 0$$

and so on.

In this manner the real grinding process is described by a sequence of alternating second and third boundary problems of thermal conductivity.

Evidently, a stationary regime will be set up some time after grinding has begun. The error in the single-application computational scheme stems from the fact that the transient process is disregarded.

To solve the boundary problem formulated above we utilized the explicit difference scheme described in [4-6]. The stability condition imposes a limitation on the permissible magnitude of the time step, which slightly increases the number of nodes of the computational mesh in comparison with other mesh methods. As far as the use of digital computers is concerned, however, this is more than offset by the great simplicity of the algorithm for computer realization of the explicit method.

The equation of thermal conductivity is replaced by the following finite-difference equation:

$$u_{i,k+1} = \left(1 - \frac{2la}{h^2}\right) u_{i,k} + \frac{la}{h^2}\ (u_{i-1,k} + u_{i+1,k}). \tag{1}$$

Remembering the condition for convergence of the method

$$l \leqslant \frac{h^2}{2a} \tag{2}$$

and taking

$$l = \frac{h^2}{4a}, \tag{3}$$

we obtain

$$u_{i,k+1} = \frac{1}{4}\,[2u_{i,k} + (u_{i+1,k} + u_{i-1,k})]. \tag{4}$$

Thus, to obtain the temperature fields in the component, the basic arithmetical operator of the method must realize formula (4) for all nodes of the computational mesh.

To simplify the calculations we go over from the case of a semi-infinite rod ($0 \leq x < \infty$) to a rod of finite length ($0 \leq x \leq H$). The quantity H we take to be the depth at which the thermal flux at the end face ($x = 0$) has no effect on the duration of the entire machining process (H is obtained from experimental data). At the end face ($x = 0$) we realize the boundary conditions corresponding to the given cycle (second kind for grinding, third kind for cooling).

The computational step h in the spatial variable we choose in accordance with the required accuracy. The step l, as previously mentioned, is determined from condition (2).

A block diagram of a program for putting the computational algorithm into effect on the Minsk-22 is given in Fig. 1.

As can be seen from the block diagram the program can be used to calculate the temperature fields produced during grinding. Some experimental data are required by way of initial values: the heat flux density q at the workpiece-tool contact; the coefficient of heat exchange during cooling α; the depth H at which the heat flux at the end face ($x = 0$) has no effect on the duration of the entire period of treatment (more precisely, on the duration of the transient regime).

The program yields not only the fields after the first pass and in the stationary regime (from which the error of the single-application scheme can be estimated) but also the fields of maximum temperatures, from which conclusions can be drawn concerning the possibility of structural changes in the workpiece.

The program was employed to calculate the temperature fields produced during the internal grinding of an alloy with a thermal conductivity $\lambda = 40$ W/m · deg.

The results of the calculations are presented in Fig. 2, which shows the temperature fields produced under various grinding conditions (for curves 1-4, 7, 8 the workpiece velocity $v_{wp} = 0.2$ m/sec, $t = 0.01$ mm; for curves 5 and 6, $v_{wp} = 0.5$ m/sec, $t = 0.01$ mm). Curves 1, 3, 5 correspond to the peak temperatures and curves 2, 4, 6 to the peak temperature gradients. Curves 3 and 4 depict the temperature field after grinding cycle I. The heat remaining in the workpiece after the cooling cycle I ($\alpha = 11{,}700$ W/m^2 · deg) gives rise to the temperature field shown by curve 8.

With succeeding grinding–cooling cycles the contact temperature increases, as does the amount of residual heat and the depth to which the workpiece is heated. Curves 1, 2, and 7 depict the thermal state of the workpiece after cycle XV; the contact temperature has increased by 105°C and the amount of accumulated heat by more than a factor of 3. This can have a considerable effect on structural changes and thermal deformations.

Increasing the speed of the workpiece helps to reduce the contact temperature and the amount of accumulated heat. For $v_{wp} = 0.5$ m/sec there is no accumulation of heat, i.e., the heat produced during a grinding cycle is completely lost during the subsequent cooling cycle. Only under such conditions is it possible to utilize the formulas derived on the assumption of a single application of the grinding tool.

An advantage of the computational technique described above is that it is readily adapted to deal with the quasi-linear formulation of the thermal conductivity problem in which allowance is made for the temperature dependence of the thermophysical constants of the material of the workpiece.

NOTATION

u is the temperature of the point with coordinate x;
t is the time of heating or cooling;

q	is the thermal flux density;
λ, a	are the thermophysical constants of the material of the workpiece;
h	is the space variable computational step;
l	is the computational time step;
α	is the heat exchange coefficient;
H	is the depth at which heat source ceases to have an effect;
v_{wp}	is the translational velocity of workpiece.

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